LIMITI NOTEVOLI
NEPERIANI

1. $\lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = \left[1^{\infty}\right] \overset{DEF}{=} e$

GENERALIZZANDO

$$\lim_{x \to \infty} \left(1 + \frac{\alpha}{\beta x}\right)^{\gamma x} = e^{\frac{\alpha}{\beta}\gamma}$$

ESEMPI

a) $\lim\limits_{x \to \infty} \left(1 + \frac{1}{3x}\right)^{2x} = e^{\frac{2}{3}}$

b) $\lim\limits_{x \to \infty} \left(1 - \frac{5}{4x}\right)^{3x} = e^{-\frac{5}{4}\cdot 3}$

2. $\lim\limits_{x \to 0} \left(1 + x\right)^{\frac{1}{x}} = \left[1^{\infty}\right] = e$

GENERALIZZANDO

$$\lim_{x \to 0} \left(1 + \alpha x\right)^{\frac{1}{\beta x}} = e^{\frac{\alpha}{\beta}}$$

ESEMPI

a) $\lim\limits_{x \to 0} \left(1 + 3x\right)^{\frac{1}{2x}} = e^{\frac{3}{2}}$

b) $\lim\limits_{x \to 0} \left(1 - \frac{5}{4}x\right)^{\frac{3}{\beta x}} = e^{-\frac{5}{4}\frac{3}{\beta}}$

FORME INDETERMINATE $[1^{\infty}]$ $[\infty^{\infty}]$

SUPER NEPERO $[1^{\infty}]$

$$\left(f(x)\right)^{g(x)} = e^{g(x)\left(f(x)-1\right)}$$

$\lim\limits_{x \to +\infty} \left(\frac{2x+3}{2x-3}\right)^{\frac{4x^2+x}{3x-1}} = \left[1^{\infty}\right] =$

$= \lim\limits_{x \to \infty} e^{\frac{4x^2+x}{3x-1} \cdot \left(\frac{2x+3}{2x-3} - 1\right)} =$

$= \lim\limits_{x \to \infty} e^{\frac{4x^2+x}{3x-1} \cdot \frac{\overbrace{\cancel{2x}+3-\cancel{2x}+3}^{6}}{2x-3}} =$

$= \lim\limits_{x \to \infty} e^{\frac{\cancel{24}^{4}\cancel{x^2}}{\cancel{6}\cancel{x^2}}} = e^{4}$

FORMA INDETERMINATA $[\infty^{0}]$

$$[f(x)]^{g(x)} = e^{\ln [f(x)]^{g(x)}} = e^{g(x) \ln f(x)}$$

ESEMPI

$\lim\limits_{x \to +\infty} (x+2)^{-\ln x} = \left[(+\infty)^{-\infty}\right] =$

$= \lim\limits_{x \to +\infty} e^{-\ln x \cdot \ln(x+2)} = \left[e^{-\infty}\right] = 0^{+}$

LIMITI NOTEVOLI
LOGARITMICI

$$\lim_{x\to 0}\frac{\log_a(1+x)}{x}=\left[\frac{0}{0}\right]=\log_a e=\frac{1}{\ln a}$$

IN PARTICOLARE

$$\lim_{x\to 0}\frac{\ln(1+x)}{x}=\left[\frac{0}{0}\right]=\log_e e=1$$

ESEMPI

a) $\displaystyle\lim_{x\to 0}\frac{\log_2(1+x)}{x}=\frac{1}{\ln 2}$

b) $\displaystyle\lim_{x\to 0}\frac{x}{\log_5(1+x)}=\ln 5$

GENERALIZZANDO

$$\lim_{x\to 0}\frac{\ln(1+\alpha x)}{\beta x}=\frac{\alpha}{\beta}$$

$$\lim_{x\to 0}\frac{\log_a(1+\alpha x)}{\beta x}=\frac{\alpha}{\beta}\log_a e=\frac{\alpha}{\beta\ln a}$$

ESEMPI

a) $\displaystyle\lim_{x\to 0}\frac{\ln(1+4x)}{5x}=\frac{4}{5}$

b) $\displaystyle\lim_{x\to 0}\frac{\log_2(1+4x)}{3x}=\frac{4}{3\ln 2}$

LIMITI NOTEVOLI
LOGARITMICI CON "TRUCCHI"

SUPER TRUCCO: COMPORTAMENTO ASINTOTIC

$x\to 0$

$$\log_a(1+\alpha x)=\alpha x\log_a e=\frac{\alpha x}{\ln a}$$

$$\ln(1+\alpha x)=\alpha x$$

$f(x)\to 0$

$$\log_a(1+f(x))=f(x)\log_a e$$

$$\ln(1+f(x))=f(x)$$

ESEMPI

a) $\displaystyle\lim_{x\to 0}\frac{\ln(1+4x)}{\ln(1+5x)}=\lim_{x\to 0}\frac{4x}{5x}=\frac{4}{5}$

b) $\displaystyle\lim_{x\to 0}\frac{\log_2(1+4x)}{\log_3(1+5x)}=\lim_{x\to 0}\frac{4x\log_2 e}{5x\log_3 e}=\frac{4\ln 3}{5\ln 2}$

TRUCCO SPECIALE

$g(x)\to 1$

$$\ln g(x)=g(x)-1$$

$$\log_a g(x)=(g(x)-1)\log_a e$$

ESEMPI

a) $\displaystyle\lim_{x\to 3}\frac{\ln(x-2)}{x^2-9}\stackrel{\frac{0}{0}}{=}\lim_{x\to 3}\frac{\overset{1}{\cancel{x-3}}}{(\cancel{x-3})(x+3)}=\frac{1}{6}$

b) $\displaystyle\lim_{x\to -2}\frac{\ln(x+3)}{x^2+2x}\stackrel{\frac{0}{0}}{=}\lim_{x\to -2}\frac{\overset{1}{\cancel{x+2}}}{x(\cancel{x+2})}=-\frac{1}{2}$

LIMITI NOTEVOLI
ESPONENZIALI

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \left[\frac{0}{0}\right] = \ln a$$

IN PARTICOLARE

$$\lim_{x \to 0} \frac{e^x - 1}{x} = \left[\frac{0}{0}\right] = 1$$

ESEMPI

a) $$\lim_{x \to 0} \frac{2^{7x} - 1}{3x} = \left[\frac{0}{0}\right] = \frac{7 \ln 2}{3}$$

b) $$\lim_{x \to 0} \frac{e^{5x} - 1}{2x} = \left[\frac{0}{0}\right] = \frac{5}{2}$$

GENERALIZZANDO

$$\lim_{x \to 0} \frac{a^{\beta x} - 1}{\gamma x} = \frac{\beta}{\gamma} \ln a$$

$$\lim_{x \to 0} \frac{e^{\beta x} - 1}{\gamma x} = \frac{\beta}{\gamma}$$

SUPER TRUCCHI

$x \to 0$

$$a^{\beta x} - 1 = \beta x \ln a$$

$$e^{\beta x} - 1 = \beta x$$

$$\lim_{x \to 0} \frac{e^{5x} - 1}{2^{3x} - 1} = \lim_{x \to 0} \frac{5x}{3x \ln 2} = \frac{5}{3 \ln 2}$$

LIMITI NOTEVOLI
GONIOMETRICI

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

GENERALIZZANDO

$$\lim_{x \to 0} \frac{\sin \alpha x}{\beta x} = \frac{\alpha}{\beta}$$

ESEMPI

a) $\lim_{x \to 0} \frac{\sin 2x}{5x} = \frac{2}{5}$

b) $\lim_{x \to 0} \frac{7x}{\sin 5x} = \frac{7}{5}$

TRUCCO: COMPORTAMENTO ASINTOTICO

$x \to 0 \qquad \sin \alpha x = \alpha x$

$f(x) \to 0 \qquad \sin f(x) = f(x)$

ESEMPI

a) $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x \to 0} \frac{2x}{3x} = \frac{2}{3}$

b) $\lim_{x \to 1} \frac{\sin(x-1)}{x^2-1} = \lim_{x \to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x+1)} = \frac{1}{2}$

LIMITI NOTEVOLI
GONIOMETRICI

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

GENERALIZZANDO

$$\lim_{x \to 0} \frac{1-\cos \alpha x}{\beta x^2} = \frac{\alpha^2}{\beta} \; \frac{1}{2}$$

ESEMPI

a) $\lim_{x \to 0} \frac{1-\cos 2x}{3x^2} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$

b) $\lim_{x \to 0} \frac{1-\cos 3x}{5x^2} = \frac{9}{5} \; \frac{1}{2} = \frac{9}{10}$

TRUCCO VELOCE

$x \to 0 \qquad 1-\cos \alpha x = \frac{1}{2} \alpha^2 x^2$

$f(x) \to 0 \qquad 1-\cos f(x) = \frac{1}{2} \; f(x)^2$

ESEMPI

$$\lim_{x \to 0} \frac{1-\cos 4x}{\sin 5x^2} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x \to 0} \frac{\frac{1}{2} 16 \cancel{x^2}}{5 \cancel{x^2}} = \frac{8}{5}$$