Studi di funzioni razionali fratte

con grafico probabile fino alla ricerca degli asintoti e con simmetrie particolari

1. $f(x)=\dfrac{x-3}{x^2-1}$ CON ASINTOTO ORIZZONTALE

2. $f(x)=\dfrac{|x|-3}{x^2-1}$ $y=f(|x|)$

3. $f(x)=\left|\dfrac{x-3}{x^2-1}\right|$ $y=|f(x)|$

4. $f(x)=\left|\dfrac{|x|-3}{x^2-1}\right|$ $y=|f(|x|)|$

5. $f(x)=\dfrac{x^2-3x+2}{x+2}$ CON ASINTOTO OBLIQUO

6. $f(x)=\dfrac{x^2-3|x|+2}{|x|+2}$

7. $f(x)=\left|\dfrac{x^2-3x+2}{x+2}\right|$

8. $f(x)=\left|\dfrac{x^2-3|x|+2}{|x|+2}\right|$

ESEMPIO STUDIO DI FUNZIONE RAZIONALE FRATTA

1) $f(x) = \dfrac{x-3}{x^2-1}$

1. RICERCA DEL DOMINIO: DENOMINATORE $\neq 0$

C.E. $x^2-1 \neq 0 \Rightarrow x \neq \pm 1$

$D = (-\infty;-1[\cup]-1;1[\cup]1;+\infty)$

2. EVENTUALI SIMMETRIE

$f(-x) = \dfrac{-x-3}{x^2-1} \neq \pm f(x)$ NE' PARI NE DISPARI

3. INTERSEZIONI CON GLI ASSI

$\cap \vec{y}$ $\begin{cases} x=0 \\ y=\dfrac{x-3}{x^2-1} \end{cases} \Rightarrow y = \dfrac{0-3}{0-1} \Rightarrow y=3$

$A(0;3)$

$\cap \vec{x}$ $\begin{cases} y=0 \quad \dfrac{N(x)}{D(x)} = 0 \Leftrightarrow N(x)=0 \\ y=\dfrac{x-3}{x^2-1} \Rightarrow \dfrac{x-3}{x^2-1}=0 \Rightarrow x-3=0 \Rightarrow x=3 \end{cases}$

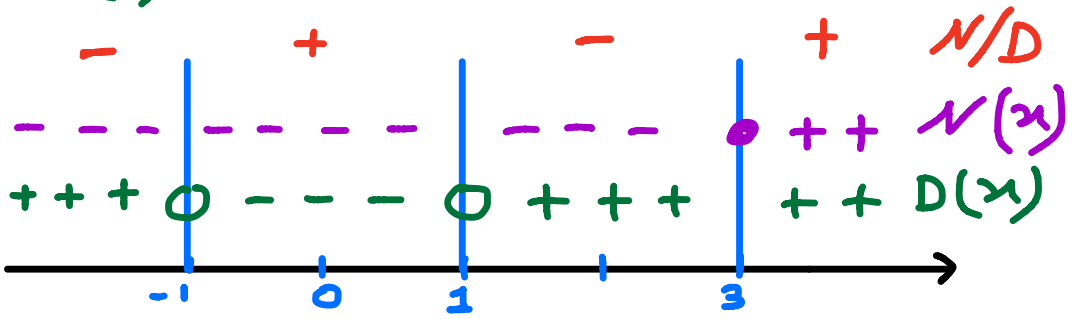
$B(3;0)$

4. STUDIO DEL SEGNO DELLA FUNZIONE

$f(x) \geq 0 \qquad \frac{x-3}{x^2-1} \geq 0$

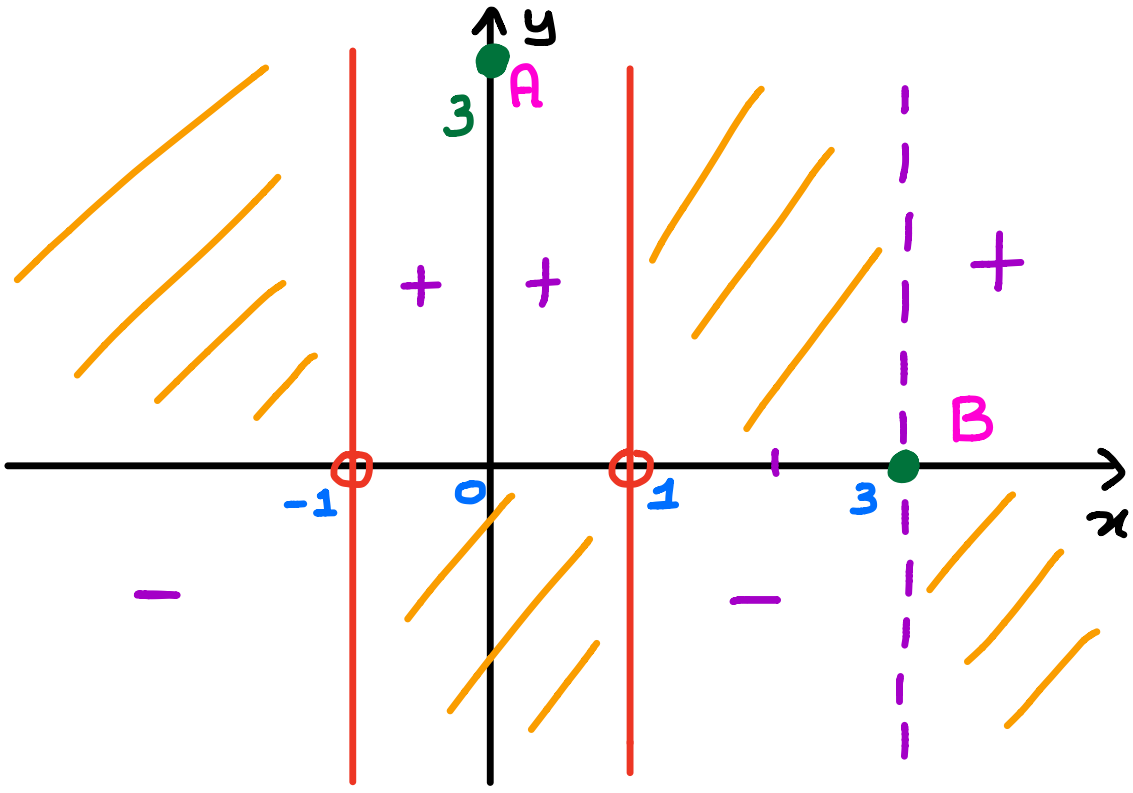
$N(x) \geq 0 \Rightarrow x-3 \geq 0 \Rightarrow x \geq 3$

$D(x) > 0 \Rightarrow x^2-1 > 0 \Rightarrow x < -1 \vee x > 1$



$f(x) \geq 0$ IN $]-1;1[\cup [3;+\infty[$

5. PRIMO APPROCCIO AL GRAFICO:



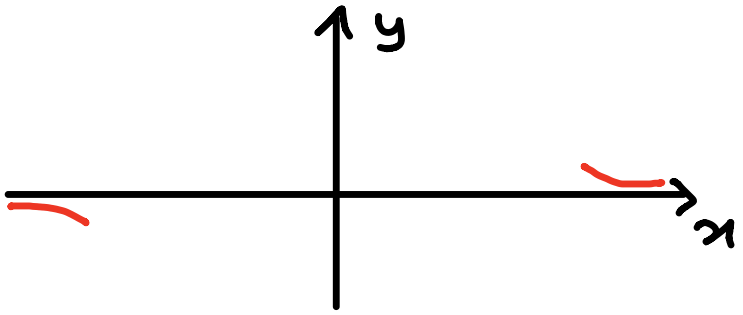
6. RICERCA DEGLI ASINTOTI ATTRAVERSO I LIMITI AGLI ESTREMI DEL C.E.

$$f(x) = \frac{x-3}{x^2-1} \qquad D =]-\infty;-1[\cup]-1;1[\cup]1;+\infty[$$

1. $\lim_{x \to -\infty} \frac{x-3}{x^2-1} = \left[\frac{\infty}{\infty}\right] = \lim_{x \to -\infty} \frac{x}{x^2} = \left[\frac{1}{-\infty}\right] \; 0^-$

2. $\lim_{x \to +\infty} \frac{x-3}{x^2-1} = \left[\frac{\infty}{\infty}\right] = \lim_{x \to -\infty} \frac{x}{x^2} = \left[\frac{1}{+\infty}\right] \; 0^+$

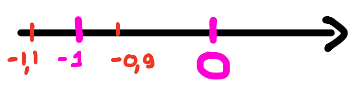
$\Rightarrow$ ASINTOTO ORIZZONTALE $y = 0$

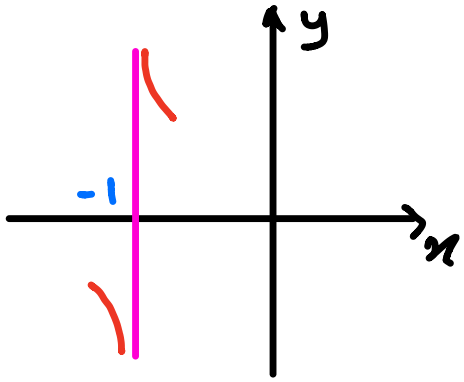


3. $\lim_{x \to -1^-} \frac{x-3}{x^2-1} = \left[\frac{-4}{1,21-1}\right] = \left[\frac{-4}{0^+}\right] = -\infty$

4. $\lim_{x \to -1^+} \frac{x-3}{x^2-1} = \left[\frac{-4}{0,9-1}\right] = \left[\frac{-4}{0^-}\right] = +\infty$

$x = -1$ ASINTOTO VERTICALE

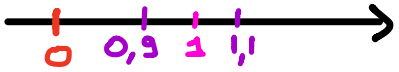


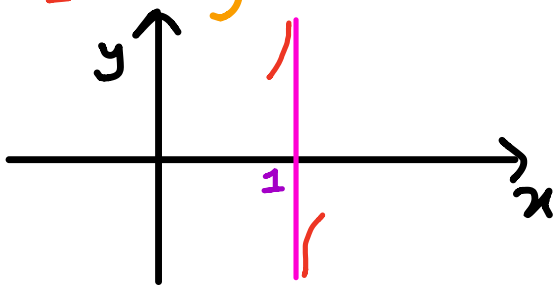


5. $\lim_{x \to 1^-} \frac{x-3}{x^2-1} = \left[\frac{-2}{0{,}9-1}\right] = \left[\frac{-2}{0^-}\right] = +\infty$

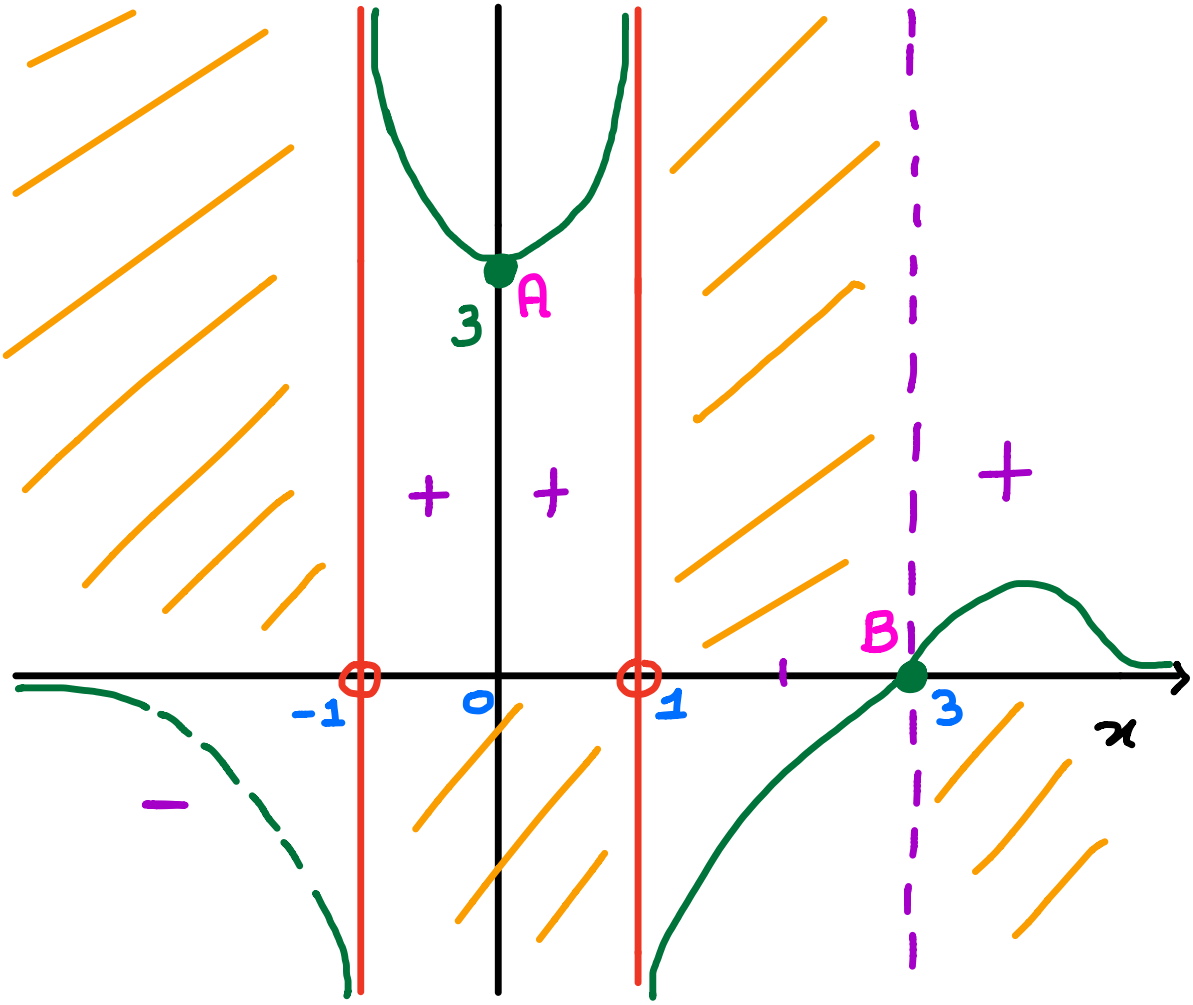
6. $\lim_{x \to 1^+} \frac{x-3}{x^2-1} = \left[\frac{-2}{1{,}1-1}\right] = \left[\frac{-2}{0^+}\right] = -\infty$

$x = 1$ ASINTOTO VERTICALE





7. GRAFICO PROBABILE



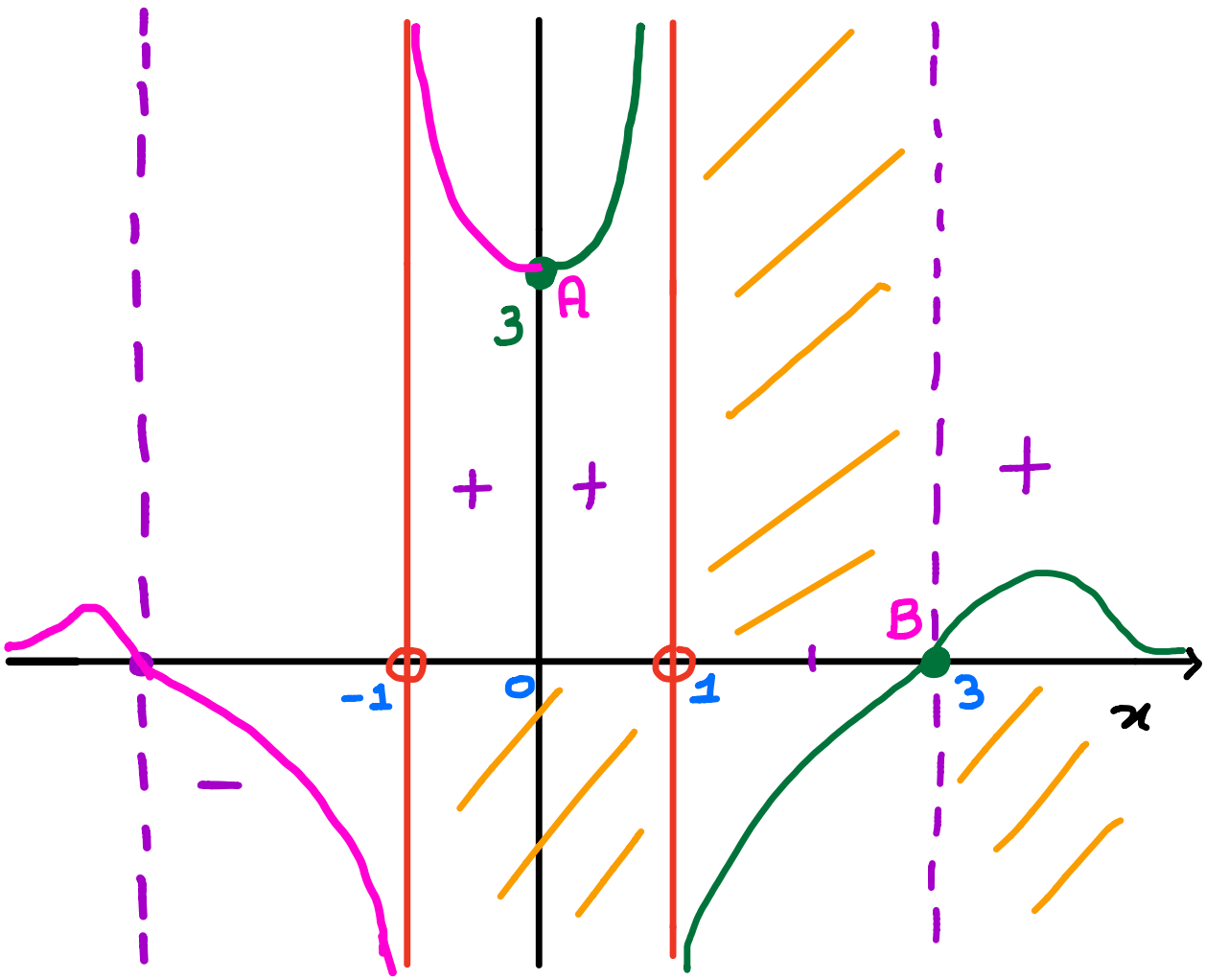
$y = f(|x|)$

2. $f(x) = \dfrac{|x| - 3}{x^2 - 1}$

SIMMETRIA PARI (ESSENDO $|-x| = |x|$)

BASTERÀ STUDIARE LA FUNZIONE PER $x \geq 0$ $\Rightarrow$ $y = \dfrac{x-3}{x^2-1}$ E QUINDI CONSIDERARE IL GRAFICO SIMMETRICO RISPETTO ALL'ASSE y (PER $x < 0$)

. GRAFICO PROBABILE

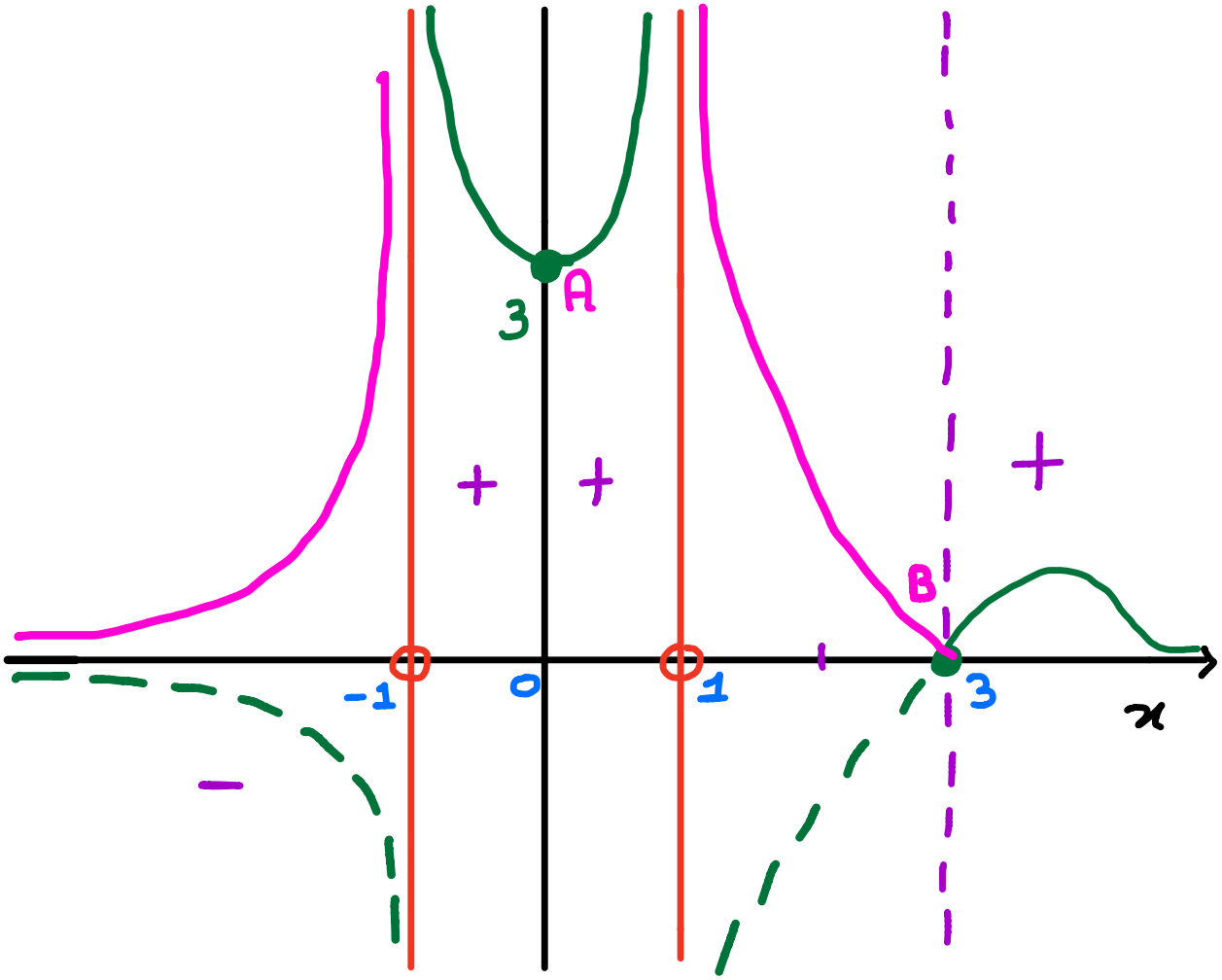


3. $f(x)=\left|\frac{x-3}{x^2-1}\right|$ $\quad y=|f(x)|$

BASTERÀ RAPRESENTARE IL GRAFICO DELLA FUNZIONE $y=\frac{x-3}{x^2-1}$

E "RIBALTARE" RISPETTO ALL'ASSE x LA PARTE NEGATIVA

. GRAFICO PROBABILE



4. $f(x)=\left|\frac{|x|-3}{x^2-1}\right|$ $\qquad y=|f(|x|)|$

SI PARTE DAL GRAFICO DELLA FUNZIONE:

$y=\frac{|x|-3}{x^2-1}$

E SI "TRASFORMA" IN POSITIVO LA PARTE NEGATIVA

. GRAFICO PROBABILE