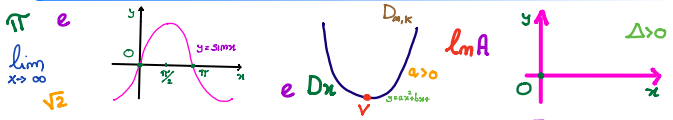
MATEMATICA A COLORI PER TUTTI

INTEGRALI INDEFINITI

FLIPPED MATH



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UNITÀ 2 SECONDO STEP
GONIOMETRICI ED ESPONENZIALI

$$\int \cos x\,dx = \sin x + c$$

$$\int \sin x\,dx = -\cos x + c$$

$$\int e^x\,dx = e^x + c$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + c$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + c$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + c$$

INTEGRALI IMMEDIATI GONIOMETRICI

1. $D\sin x = \cos x \Rightarrow \int \cos x\,dx = \sin x + c$

2. $D\cos x = -\sin x \Rightarrow \int \sin x\,dx = -\cos x + c$

ESEMPIO

$$\int 2\sin x - 3\cos x\,dx =$$

$$= 2\int \sin x\,dx - 3\int \cos x\,dx =$$

$$= -2\cos x - 3\sin x + c$$

3. $D\tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

$$\Rightarrow \int \frac{1}{\cos^2 x}\,dx = \tan x + c$$

SI AGGIUNGE E SI SOTTRAE 1

$$\int \tan^2 x\,dx = \int \left(\tan^2 x + 1\right) - 1\,dx =$$

$$= \tan x - x + c$$

4. $D\cot x = -\frac{1}{\sin^2 x} = -\left(1+\cot^2 x\right)$

$$\Rightarrow \int \frac{1}{\sin^2 x}\,dx = -\cot x + c$$

$$\Rightarrow \int \cot^2 x\,dx = \int \left(\cot^2 x + 1\right) - 1\,dx =$$

$$= -\cot x - x + c$$

ESEMPI

1. $$\int \frac{3}{2\sin^2 x} - \frac{4}{5\cos^2 x}\,dx =$$

$$= \frac{3}{2}\int \frac{1}{\sin^2 x}\,dx - \frac{4}{5}\int \frac{1}{\cos^2 x}\,dx =$$

$$= -\frac{3}{2}\cot x - \frac{4}{5}\tan x + c$$

2. $\int 2\tan^2 x - 2\cot^2 x \, dx =$

$= 2\int \left(\tan^2 x + 1\right) \underbrace{- 1 - \cot^2 x}_{-\left(\cot^2 x + 1\right)} \, dx =$

$= 2\left[\tan x + \cot x\right] + c =$

$= 2\tan x + 2\cot x + c$

ANCORA ALTRI 2...

5. $D\arcsin x = \dfrac{1}{\sqrt{1-x^2}}$

6. $D\arccos x = -\dfrac{1}{\sqrt{1-x^2}}$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + c$$

$$D\arctan x = \frac{1}{1+x^2}$$

$$D\operatorname{arccot} x = -\frac{1}{1+x^2}$$

$$\Rightarrow \boxed{\int \frac{1}{1+x^2}\,dx = \arctan x + c}$$

ESEMPI

1. $$\int \frac{3}{2\sqrt{1-x^2}}\,dx = \frac{3}{2}\arcsin x + c$$

2. $$\int \frac{5}{4x^2+4}\,dx = \frac{5}{4}\int \frac{1}{1+x^2}\,dx =$$

$$= \frac{5}{4}\arctan x + c$$

ATTENZIONE!

NON SIAMO ANCORA PRONTI A RISOLVERE INTEGRALI DEL TIPO:

$$\int \frac{1}{1+4x^2}\,dx \qquad \int \frac{x}{\sqrt{1-9x^2}}\,dx \qquad ..$$

INTEGRALI IMMEDIATI ESPONENZIALI

$D e^x = e^x \Rightarrow \int e^x \, dx = e^x + c$

$D a^x = a^x \ln a \Rightarrow \int a^x \, dx = \frac{a^x}{\ln a} + c$

ESEMPI

1. $\int 2^x \, dx = \frac{2^x}{\ln 2} + c$

2. $\int 3^{x+1} \, dx = \int 3^x \cdot 3 \, dx =$

$= 3 \cdot \frac{3^x}{\ln 3} + c = \frac{3^{x+1}}{\ln 3} + c$

N.B. QUESTO INTEGRALE LO RISOLVEREMO MEGLIO CON IL METODO DEGLI INTEGRALI DI FUNZIONI COMPOSTE!

2. $\int \frac{4^{x+1}}{3^{2x-1}} \, dx = \int \frac{4^x \cdot 4}{3^{2x} \cdot 3^{-1}} \, dx =$

$3^{2x} = 9^x$

PROVIAMO A SCOMPORRE LE POTENZE

$$= \frac{4}{3^{-1}} \int \underbrace{\left(\frac{4}{9}\right)^x}_{\hookrightarrow \frac{4^x}{9^x}} dx = \frac{4}{3^{-1}} \left(\frac{4}{9}\right)^x \frac{1}{\ln \frac{4}{9}} + C =$$

$$= \frac{4^{x+1}}{3^{2x-1}} \frac{1}{\ln \frac{4}{9}} + C$$

RICORDIAMO CHE...

$$\ln \frac{4}{9} = \ln 4 - \ln 9 = \ln 2^2 - \ln 3^2$$

$$= 2\ln 2 - 2\ln 3$$

IL RISULTATO SI POTEVA INTUIRE

..

PROVA A VERIFICARE CHE...

$$\int \frac{25^{x-2}}{7^{2x+3}} dx = \frac{25^{x-2}}{7^{2x+3}} \frac{1}{\ln \frac{25}{49}} + C$$