Color est e pluribus unus

corso di Matematica

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Modulo 3: Limiti notevoli
Funzioni reali di variabile reale

Unita' 4:
Ultimi limiti notevoli

- **Ultimo Limite notevole**
- **Super Nepero**

"Non accontentarti di restare nel GRIGIO per paura
del NERO, ma punta dritto al BIANCO..
e tuffati dentro!
Entra nel vortice.. quindi, rallenta:
ritroverai tutti i COLORI
e farai splendere sempre la tua Vita!!"

ULTIMO LIMITE NOTEVOLE

$$\lim_{x\to 0} \frac{(1+x)^{\alpha}-1}{x} = \left[\frac{0}{0}\right] = \alpha$$

GENERALIZZAZIONI

$$\lim_{x\to 0} \frac{(1+\gamma x)^{\alpha}-1}{\beta x} = \alpha\,\frac{\gamma}{\beta}$$

$$\lim_{f(x)\to 0} \frac{(1+f(x))^{\alpha}-1}{f(x)} = \alpha$$

PROVACI SUBITO TU...

1) $$\lim_{x\to 0} \frac{(1+3x)^{2}-1}{4x} = \frac{3}{2}$$

2) $$\lim_{x\to 0} \frac{(1+\sin 2x)^{3}-1}{4x} = \frac{3}{2}$$

3) $$\lim_{x\to 0} \frac{(1+3\tan 2x)^{\frac{1}{2}}-1}{\ln(1-\sin 4x)} = -\frac{3}{4}$$

OSSERVAZIONE

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

EX. $$\lim_{x \to 0} \frac{\sqrt[2]{(1+x)^1} - 1}{x} = \left[\frac{0}{0}\right] =$$

$$\lim_{x \to 0} \frac{(1+x)^{1/2} - 1}{x} = \frac{1}{2}$$

PROVACI TU...

1) $$\lim_{x \to 0} \frac{\sqrt[3]{1-2x} - 1}{\sin 3x} = -\frac{2}{9}$$

2) $$\lim_{x \to 0} \frac{1 - \sqrt[4]{(1-3x)^2}}{\ln\left(1 - 3\tan 2x\right)}$$

3) $$\lim_{x \to 0} \frac{1 - \cos 5x}{\sqrt[3]{(1-2\sin 3x)^2} - e^{4x}} =$$

COMPORTAMENTO ASINTOTICO

$x \to 0 \qquad (1+\beta x)^{\alpha} - 1 = \alpha \beta x$

$f(x) \to 0 \qquad (1+f(x))^{\alpha} - 1 = \alpha f(x)$

PROVACI TU... SUBITO!

1) $$\lim_{x \to 0} \frac{(1-5x)^3 - 1}{1 - \sqrt{1+\sin 2x}} = 15$$

2) $$\lim_{x \to 2^+} \frac{\sqrt{1+\ln(x-1)} - 1}{3\tan(x^2-4)} =$$

CASO INTERESSANTE..

$g(x) \to 1 \qquad (g(x))^{\alpha} - 1 = \alpha \cdot (g(x) - 1)$

EX

$$\lim_{x \to 3} \frac{\sqrt[2]{\overbrace{x-2}^{\to 1}} - 1}{\ln\underbrace{(x^2-8)}_{1}} = \lim_{x \to 3} \frac{\frac{1}{2}\left[(x-2)-1\right]}{(x^2-8)-1} = [\ldots]$$

PROVACI TU...

1) $$\lim_{x \to -2} \frac{\sqrt{x+3} - e^{x+2}}{\ln(x^2-3)} = +\frac{1}{8}$$

2) $$\lim_{x \to 4} \frac{\tan(4-x)}{1-\sqrt{x-3}} =$$

SUPER NEPERO 1^{∞}

$$\left(f(x)\right)^{g(x)} = e^{g(x)\left(f(x)-1\right)}$$

EX. $$\lim_{x \to +\infty} \left(1+2x\right)^{e^x} = \left[1^{\infty}\right] =$$

$$= \lim_{x \to +\infty} e^{e^x \cdot (1+2x-1)}$$

$$= \lim_{x \to +\infty} e^{2x \cdot e^x} = \left[e^{+\infty}\right] = +\infty$$

EX 2 $\lim_{x \to +\infty} \left(\frac{2x+3}{2x-3}\right)^{\frac{4x^2+x}{3x-1}} = [1^\infty] =$

$$= \lim_{x\to\infty} e^{\frac{4x^2+x}{3x-1}\cdot\left(\frac{2x+3}{2x-3}-1\right)} =$$

$$= \lim_{x\to\infty} e^{\frac{4x^2+x}{3x-1}\cdot\frac{\overbrace{\cancel{2x}+3-\cancel{2x}+3}^{6}}{2x-3}} =$$

$$= \lim_{x\to\infty} e^{\frac{\cancel{24}^{4}\cancel{x^2}}{\cancel{6}\cancel{x^2}}} = e^4$$

PROVACI TU...

1) $\lim_{x\to 0^+} (1+\ln x)^{\frac{2}{x^2}} = ?$

2) $\lim_{x\to +\infty} \left(\frac{3x^2-5x+2}{3x^2+5x-2}\right)^{\frac{2x^2-x+1}{4x-5}} =$

ULTIMA FORMA INDETERMINATA $[\infty^{\infty}]$

SI RISOLVE TRAMITE LA FORMULA

$$[f(x)]^{g(x)} = e^{\ln [f(x)]^{g(x)}} = e^{g(x)\ln f(x)}$$

EX.1 $$\lim_{x \to +\infty} (x+2)^{-\ln x} = \left[(+\infty)^{-\infty}\right] =$$

$$= \lim_{x \to +\infty} e^{-\ln x \cdot \ln(x+2)} = \left[e^{-\infty}\right] = 0^{+}$$

EX 2 $$\lim_{x \to 2^{+}} [1-\ln(x-2)]^{\frac{1}{\underbrace{\sin(x-2)}_{x-2}}} = \left[(+\infty)^{\infty}\right]$$

$$= \lim_{x \to 2^{+}} [1-\ln(x-2)]^{\frac{1}{x-2}} =$$

$$= \lim_{x \to 2^{+}} e^{\frac{1}{x-2} \ln[1-\ln(x-2)]} = e^{\left[\frac{+\infty}{0^{+}}\right]} = e^{+\infty} =$$

$$= +\infty$$

PROVACI TU...

1) $\lim_{x\to+\infty} (x^2-1)^{2x} =$

2) $\lim_{x\to+\infty} (3+e^x)^{x-2} =$

3) $\lim_{x\to+\infty} (x^2-x)^{8-x^3} =$

4) $\lim_{x\to 0^+} \left(1+\ln(1+2\sin 3x)\right)^{\frac{e^{\tan 3x}-1}{1-\cos 2x}}$

5) $\lim_{x\to 0^+} \left(1+\tan 4x\right)^{\frac{\ln\cos x}{1-\sqrt{1-\sin x^3}}}$

6) $\lim_{x\to 0} \left(\frac{4x^2+4x+1}{4x^2-4x+1}\right)^{\frac{\sqrt{1+3\sin x}-1}{e^{2x^2}-\cos 4x}} =$

7) $\lim_{x\to 3^+} \left(1-\ln(x-2)\right)^{\frac{1}{\sqrt{3x-8}-1}} =$