

Color est e pluribus unus

corso di Matematica

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Modulo 3: Limiti notevoli

Funzioni reali di variabile reale

Unita' 5: esercitazione

limiti notevoli

- Limite notevole neperiano
- Limiti notevoli logaritmici ed esponenziali
- Limiti notevoli goniometri i

"Non accontentarti di restare nel GRIGIO per paura
del NERO, ma punta dritto al BIANCO..
e tuffati dentro!

Entra nel vortice.. quindi, rallenta:

ritroverai tutti i COLORI

e farai splendere sempre la tua Vita!!"

Esercitazione limiti notevoli livello 1

$$1. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^{5x} = e^2 \sqrt{e}$$

$$2. \lim_{x \rightarrow -\infty} \left(1 - \frac{3}{2x}\right)^{4x} = \frac{1}{e^6}$$

$$3. \lim_{x \rightarrow 0} \left(1 + 5x\right)^{\frac{1}{2x}} = e^2 \sqrt{e}$$

$$4. \lim_{x \rightarrow 0} \left(1 - 3x\right)^{\frac{1}{5x}} = \sqrt[5]{e^3}$$

$$5. \lim_{x \rightarrow 0} \frac{\ln(1+3x)}{4x} = \frac{3}{4}$$

$$6. \lim_{x \rightarrow 0} \frac{\log_2(1+x)}{2x} = \frac{1}{2 \ln 2} = \frac{1}{\ln 4}$$

$$7. \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{5x} = \frac{2}{5}$$

$$8. \lim_{x \rightarrow 0} \frac{3^{2x} - 1}{4x} = \frac{1}{2} \ln 3 = \ln \sqrt{3}$$

$$9. \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = \frac{2}{2}$$

$$10. \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{3x} = 0$$

$$11. \lim_{x \rightarrow 0} \frac{1 - \cos 5x}{3x^2} = \frac{25}{6}$$

Esercitazione limiti notevoli livello 2

$$1. \lim_{x \rightarrow \infty} \left(1 - \frac{3}{2x}\right)^{\frac{2}{5}x} = \sqrt[5]{e^3}$$

$$2. \lim_{x \rightarrow -\infty} \left(\frac{5x-1}{5x+1}\right)^{4x} = e^{-\frac{8}{5}}$$

$$3. \lim_{x \rightarrow 0} \left(1 - \frac{3}{4}x\right)^{\frac{x}{2x}} = e^{-\frac{21}{8}}$$

$$4. \lim_{x \rightarrow 0^-} (2-3x)^{\frac{1}{5x}} = 0^+$$

$$5. \lim_{x \rightarrow 0} \frac{\ln(1+3x)}{\ln(1-2x)} = -\frac{3}{2}$$

$$6. \lim_{x \rightarrow 0^+} \frac{\log_2(1+x)}{\log_3(1-x^2)} = -\infty$$

$$7. \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{e^{3x} - 1} = \frac{2}{3}$$

$$8. \lim_{x \rightarrow 0} \frac{3^{2x} - 1}{2^{3x} - 1} = \frac{2 \ln 3}{3 \ln 2} = \log_8 9$$

$$9. \lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 5x} = \frac{2}{5}$$

$$10. \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{\sin 2x} = 0$$

$$11. \lim_{x \rightarrow 0} \frac{1 - \cos 5x}{\sin 3x^2} = \frac{25}{6}$$

Esercitazione limiti notevoli livello 3

$$1. \lim_{x \rightarrow -\infty} \left(\frac{x^2 - 3x + 1}{2x^2 + 3x} \right)^{\frac{x^2 + 1}{2x - 1}} = e^{-\frac{3}{2}} \quad (?)$$

$$2. \lim_{x \rightarrow -\infty} \left(\frac{5x - 1}{6x + 1} \right)^{4x} = +\infty$$

$$3. \lim_{x \rightarrow -\infty} \left(\frac{5x - 1}{5x + 1} \right)^{\frac{3x + 1}{2x - 1}} = 1$$

$$4. \lim_{x \rightarrow 0} \left(1 + \sin x \right)^{\frac{1}{5x}} = \sqrt[5]{e}$$

$$5. \lim_{x \rightarrow 0^+} \frac{\ln(1 + 3x^2)}{\ln(1 - 2x)} = 0^-$$

$$6. \lim_{x \rightarrow +\infty} \frac{\ln(1 + x)}{e^{2x} - 1} = 0^+$$

$$7. \lim_{x \rightarrow -\infty} \frac{e^{2x} - 1}{e^{3x} - 1} = 1$$

$$8. \lim_{x \rightarrow 0^+} \frac{3^{2x} - 1}{2^{3x^2} - 1} = +\infty$$

$$9. \lim_{x \rightarrow 0^+} \frac{\sin 2x^2}{\sin 5x} = 0^+$$

$$10. \lim_{x \rightarrow 0^-} \frac{1 - \cos 4x}{\sin 2x^3} = -\infty$$

$$11. \lim_{x \rightarrow 0} \frac{\cos 5x - 1}{4 \sin 3x^2} = -\frac{25}{24}$$

Esercitazione limiti notevoli alto livello 1

$$1) \lim_{x \rightarrow 0} \frac{5 \sin 3x + 1 - \cos 4x}{4x^2 - 2 \tan 5x} = -\frac{3}{2} \quad (*)$$

VEDI

SVOLGIMENTO

$$2) \lim_{x \rightarrow 0} \frac{\ln(1 - 3 \sin x) + 4 \tan 2x^2}{e^{3x} - \cos 2x - 3x} = \infty \quad (*)$$

$$3) \lim_{x \rightarrow 0} \frac{\cos 5x - 1 + 2 \sin x^2}{\ln(1 - \tan^2 x)} = \frac{21}{2} \quad (*)$$

$$6) \lim_{x \rightarrow 0} (1 - \sin 3x)^{\frac{\ln(\cos x)}{x \tan x^2}} = \infty \quad (*)$$

$$4) \lim_{x \rightarrow 0} \left(1 - \ln(1 + 4 \tan 3x)\right)^{\frac{e^{\sin x} - 1}{1 - \cos 2x}} = \frac{1}{e^6} \quad (*)$$

$$3^*) \lim_{x \rightarrow 0} (1 + 2 \tan 3x)^{\frac{1}{\sin 4x}} = e\sqrt{e}$$

Alcuni svolgimenti (contributo ex studenti)

① $\lim_{x \rightarrow 0} \frac{\ln(1-3\sin x)}{e^{\tan 4x} - 1} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 0} \frac{\ln(1-3x)}{e^{4x} - 1} = \lim_{x \rightarrow 0} \frac{-3x}{4x} = -\frac{3}{4}$

② $\lim_{x \rightarrow 0} \frac{e^{2\tan^2 x} - \cos 3x}{\ln(1-4\tan^2 3x)} = \lim_{x \rightarrow 0} \frac{e^{2x^2} - \cos 3x}{\ln(1-12x^2)} =$
 $= \lim_{x \rightarrow 0} \frac{(e^{2x^2} - 1) + (-\cos 3x)}{-12x^2} = \frac{2x^2 + \frac{9}{2}x^2}{-12x^2} = \frac{\frac{13}{2}}{-12} = -\frac{13}{24}$

③ $\lim_{x \rightarrow 0} (1+2\tan 3x)^{\frac{1}{\sin 4x}} = [1^\infty] =$
 $= \lim_{x \rightarrow 0} (1+6x)^{\frac{1}{4x}} = e^{\frac{6}{4}} = e^{\frac{3}{2}}$ **3***

④ $\lim_{x \rightarrow 0} (1 - \ln(1+4\tan 3x))^{\frac{e^{\sin x} - 1}{1 - \cos 2x}} =$
 $= \lim_{x \rightarrow 0} (1 - \ln(1+12x))^{\frac{e^x - 1}{\frac{1}{2}4x^2}} =$
 $= \lim_{x \rightarrow 0} (1 - 12x)^{\frac{x}{2x^2}} = [1^\infty] =$ **4**
 $= e^{-\frac{12}{2}} = e^{-6}$

⑤ $\lim_{x \rightarrow 0^+} \frac{\sqrt{1-\cos 4x}}{\ln(1+3\sin 2x^2)} = \left[\frac{0}{0} \right] = \frac{\sqrt{\frac{1}{2}16x^2}}{\ln(1+6x^2)} =$
 $= \lim_{x \rightarrow 0^+} \frac{2\sqrt{2}x}{6x^2} = +\infty$

① $\lim_{x \rightarrow 0} \frac{5\sin 3x + 1 - \cos 4x}{4x^2 - 2\tan 5x} = \lim_{x \rightarrow 0} \frac{15x + \frac{1}{2}16x^2}{4x^2 - 10x} = \left[\frac{0}{0} \right]$

$\lim_{x \rightarrow 0} \frac{x(15+8x)}{x(4x-10)} = -\frac{15}{10} = -\frac{3}{2}$ **1**

② $\lim_{x \rightarrow 0} \frac{\ln(1-3\sin x) + 4\tan 2x^2}{e^{3x} - \cos 2x - 3x} =$
 $= \lim_{x \rightarrow 0} \frac{\ln(1-3x) + 8x^2}{(e^{3x}-1) + (-\cos 2x) - 3x} =$ **2**
 $= \lim_{x \rightarrow 0} \frac{-3x + 8x^2}{3x + \frac{1}{2}4x^2 - 3x} =$
 $= \lim_{x \rightarrow 0} \frac{x(8x-3)}{2x^2} = \left[\frac{-3}{0} \right] = \infty$

③ $\lim_{x \rightarrow 0} \frac{\cos 5x - 1 + 2\sin x^2}{\ln(1-\tan^2 x)} =$ **3**
 $= \lim_{x \rightarrow 0} \frac{-\frac{1}{2}25x^2 + 2x^2}{-x^2} = -\left(\frac{-25+4}{2}\right) = \frac{21}{2}$

SUPER TRUCCHI

6) $\lim_{x \rightarrow 0} (1 - \sin 3x)^{\frac{\ln(\cos x)}{x \tan x^2}} = \lim_{x \rightarrow 0} (1 - 3x)^{\frac{\ln(\cos x)}{x \tan x^2}}$

$\lim_{x \rightarrow 0} (1 - 3x)^{-\frac{1}{2x}} = e^{\frac{3}{2}} = e\sqrt{e}$

$\frac{\cos x - 1}{x \tan x^2} \sim -\frac{1}{2} \frac{1}{x^2}$

Esercitazione limiti notevoli alto livello

$$1) \lim_{x \rightarrow 0} \frac{\ln(1 - 2\sin 3x)}{\ln(1 + 3\tan 2x)} = -1$$

$$2) \lim_{x \rightarrow 3} \frac{\ln(2x - 5)}{\sqrt{x^2 - 8} - 1} = \frac{2}{3}$$

$$3) \lim_{x \rightarrow +\infty} \left(\frac{5x^2 - 3x - 1}{5x^2 + 3x + 1} \right)^{\frac{\sin \frac{1}{x}}{e^{\frac{2}{x^2}} - 1}} = e^{-\frac{6}{5}}$$

$$4) \lim_{x \rightarrow 0} \frac{e^{2\tan^2 x} - \cos 4x}{x \tan 3x - \ln(1 - \sin^2 2x)} = \frac{10}{7}$$

ATTENZIONE AI FINITI LIMITI NOTEVOLI

$$1) \lim_{x \rightarrow 0} \frac{e^{3 \tan x} + \cos 2x}{\ln(1 - 2 \sin^2 x)} = \infty$$

$$1b) \lim_{x \rightarrow 0} \frac{e^{3 \tan x} - \cos 2x}{\ln(2 + \sin^2 x)} = 0$$

$$1c) \lim_{x \rightarrow 0} \frac{e^{3 \tan x} - \cos 2x}{\ln(1 - 2 \sin^2 x)} =$$

$$2a) \lim_{x \rightarrow +\infty} \left(\frac{3x^2 - 2x + 1}{3x^2 + 2x - 1} \right)^{\frac{5x^2 - 1}{5x + 1}} =$$

$$2b) \lim_{x \rightarrow +\infty} \left(\frac{3x^2 - 2x + 1}{3x^2 + 2x - 1} \right)^{\frac{5x - 1}{5x^2 + 1}} = 1$$

$$2c) \lim_{x \rightarrow +\infty} \left(\frac{3x^2 - 2x + 1}{4x^2 + 2x - 1} \right)^{\frac{5x^2 - 1}{5x + 1}} = 0^+$$

$$2d) \lim_{x \rightarrow 0} \left(\frac{3x^2 - 2x + 1}{3x^2 + 2x - 1} \right)^{\frac{5x^2 - 1}{5x + 1}} = (?)$$

$$3a) \lim_{x \rightarrow -2^+} \frac{\ln(x^2 - 3) + e^{x+2}}{\cos(x+2) - 1} = -\infty$$

$$3b) \lim_{x \rightarrow -2^-} \frac{\ln(x^2 + 3) - e^{x+2}}{\cos(x+2) - 1} = -\infty$$

$$3c) \lim_{x \rightarrow -2} \frac{\ln(x^2 - 3) - e^{x+2}}{\cos(x+2) - 1} = (?)$$