INTEGRALI PER SOSTITUZIONE

FORMULA DI RIFERIMENTO

$$\int g(f(x))\, df(x) = \int g(t)\, dt$$

ESERCIZI E SOSTITUZIONI NOTEVOLI

1. SE ABBIAMO x DENTRO E FUORI LA RADICE

ESEMPIO

$$\int \frac{x-2}{\sqrt{x+1}}\, dx = *$$

SI PONE $\sqrt{x+1} = t$ DA CUI

$x+1 = t^2 \Rightarrow x = t^2-1 \Rightarrow dx = 2t\,dt$

$$* = \int \frac{t^2-1-2}{\cancel{t}}\, 2\cancel{t}\, dt = 2\int t^2-3\, dt =$$

$$= 2\frac{t^3}{3} - 6t + c = \frac{2}{3}\sqrt{(x+1)^3} - 6\sqrt{x+1} + c$$

2. UN INTEGRALE NOTEVOLE

$$\int \cos^2 x \, dx$$

RICHIAMANDO LA FORMULA DI BISEZIONE DEL COSENO:

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

PONENDO $\frac{\alpha}{2} = x$ SI HA:

$$\cos^2 x = \frac{1+\cos 2x}{2}$$ QUINDI

$$\int \cos^2 x \, dx = \int \frac{1+\cos 2x}{2} \, dx =$$

$$= \int \frac{1}{2} \, dx + \frac{1}{2 \cdot 2}\int \cos \underbrace{2x}_{} \, 2\, dx =$$

$f(x) = 2x \quad df(x) = 2\, dx$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x + c$$

PROVACI TU.. $\int \sin^2 x \, dx$

3.

$$\int \sqrt{a-bx^2}\, dx \qquad x=\sqrt{\frac{a}{b}}\sin t$$

$$\int \sqrt{9-16x^2}\, dx = \qquad x=\sqrt{\frac{9}{16}}\sin t = \frac{3}{4}\sin t$$

$$= * \qquad dx = \frac{3}{4}\cos t\, dt$$

$$9-16x^2 = 9-\cancel{16}\cdot\frac{9}{\cancel{16}}\sin^2 t = 9(1-\sin^2 t) =$$

$$= 9\cos^2 t$$

$$* = \int \underbrace{\sqrt{9\cos^2 t}}_{3}\ \frac{3}{4}\cos t\, dt =$$

$$= 3\ \frac{3}{4}\underbrace{\int \cos^2 t\, dt}_{\text{DALL' ESEMPIO PRECEDENTE}} = \frac{9}{4}\left[\frac{1}{2}t+\frac{1}{4}\sin 2t\right]+c$$

$$\cos^2 t = \frac{1+\cos 2t}{2}$$

$$\int \cos^2 t\, dt = \frac{1}{2}t+\frac{1}{4}\sin 2t + c$$

ESSENDO

$$x = \frac{3}{4}\sin t \rightarrow \sin t = \frac{4}{3}x$$

$$t = \arcsin\frac{4}{3}x$$

DOBBIAMO ESPRIMERE $\sin 2t$ IN FUNZIONE DI $\sin t$:

$$\sin 2t = 2\sin t \cos t = 2\sin t\sqrt{1-\sin^2 t} =$$

$$\sin t = \frac{4}{3}x$$

$$= 2\cdot\frac{4}{3}x\sqrt{1-\frac{16}{9}x^2} = \frac{8}{3}x\sqrt{\frac{9-16x^2}{9}} =$$

$$= \frac{8}{9}x\sqrt{9-16x^2}$$

AVEVAMO TROVATO

$$\int \cos^2 t\, dt = \frac{1}{2}t + \frac{1}{4}\sin 2t =$$

$$= \frac{1}{2}\arcsin\frac{4}{3}x + \frac{1}{\cancel{4}}\,\frac{\overset{2}{\cancel{8}}}{9}x\sqrt{9-16x^2} + C$$

MOLTIPLICANDO PER $\frac{9}{4}$ SI HA :

$$* = \frac{9}{8}\arcsin\frac{4}{3}x + \frac{1}{2}x\sqrt{9-16x^2} + C$$

PROVACI TU...

1. $\int \frac{x}{1+\sqrt{x}}\,dx = \frac{2}{3}x\sqrt{x} - x + 2\sqrt{x} - \ln(1+\sqrt{x})^2 + c$

2. $\int \frac{1+x}{1+\sqrt{x}}\,dx = \frac{2}{3}x\sqrt{x} - x + 4\sqrt{x} - \ln(1+\sqrt{x})^4 + c$

3. $\int \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}\,dx = \arcsin\sqrt{x} + c$

4. $\int \frac{1}{1+\sqrt{x}}\,dx = 2\sqrt{x} - 2\ln(1+\sqrt{x}) + c$

5. $\int \sqrt{16-x^2}\,dx = 8\arcsin\frac{x}{4} + \frac{x}{2}\sqrt{16-x^2} + c$

6. $\int \sqrt{3-x^2}\,dx = \frac{3}{2}\arcsin\frac{x}{\sqrt{3}} + \frac{x}{2}\sqrt{3-x^2} + c$

7. $\int \sqrt{1-5x^2}\,dx = \frac{1}{2\sqrt{5}}\arcsin x\sqrt{5} + \frac{x}{2}\sqrt{1-5x^2} + c$

8 $\int \sqrt{2-3x^2}\,dx = \frac{\sqrt{3}}{3}\arcsin x\sqrt{\frac{3}{2}} + \frac{x}{2}\sqrt{2-3x^2} + c$

4. GONIOMETRICI IN $\sin x$ $\cos x$

$$\int \sin^3 x \cos x \, dx = \int (\sin x)^3 \cos x \, dx =$$

$\sin x = t$ $\quad\quad$ $\cos x \, dx = dt$

$$= \int t^3 \, dt = \frac{t^4}{4} + c = \frac{1}{4} \sin^4 x + c$$

$$\int \underbrace{\sin^3 x}_{\uparrow} \cos^2 x \, dx = *$$

SI LAVORA CON QUELLO CHE HA POTENZA DISPARI

$$\sin^3 x \cos^2 x = \sin x \cdot \underbrace{\sin^2 x}_{1-\cos^2 x} \cdot \cos^2 x =$$

$$= \sin x \cdot (\cos^2 x - \cos^4 x)$$

SI PONE: $\cos x = t$ $\quad\quad$ $\sin x \, dx = \underset{\uparrow}{-} dt$

$D \cos x = - \sin x$

$$* = - \int t^2 - t^4 \, dt =$$

$$= -\frac{t^3}{3} + \frac{t^5}{5} + c = \frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + c$$

5. TANGENTE

$$\int \tan^3 x\, dx = \qquad \tan x = t$$

$$x = \arctan t$$

$$dx = \frac{1}{1+t^2}\, dt$$

$$= \int t^3 \frac{1}{1+t^2}\, dt = \int \frac{t^3}{t^2+1}\, dt = *$$

DIVISIONE TRA POLINOMI

$$\begin{array}{rrrr|l} t^3 & +0t^2 & +0t & +0 & t^2+1 \\ \cline{5-5} -t^3 & & -t & & t \\ \hline = & = & -t & & \end{array}$$

$$* = \int t - \frac{t}{1+t^2}\, dt =$$

$$= \int t\, dt - \frac{1}{2}\int \frac{2t}{1+t^2}\, dt = \frac{t^2}{2} - \frac{1}{2}\ln|1+t^2| + c =$$

$$= \frac{\tan^2 x}{2} - \frac{1}{2}\ln(1+\tan^2 x) + c$$

PROVACI TU

$$\int \tan^4 x\, dx = \frac{\tan^3 x}{3} - \tan x + x$$

6. ANCORA IN $\sin x$ $\cos x$

$$\int \frac{1}{\sin x}\,dx = *$$

QUESTA VOLTA SI USANO LE FORMULE PARAMETRICHE

$$\sin x = \frac{2t}{1+t^2} \qquad t = \tan\frac{x}{2}$$

$$x = 2\arctan t$$

$$dx = \frac{2}{1+t^2}\,dt$$

$$* = \int \frac{\cancel{1+t^2}}{\cancel{2}t}\,\frac{\cancel{2}}{\cancel{1+t^2}}\,dt = \ln|t| + c =$$

$$= \ln\left|\tan\frac{x}{2}\right| + c$$

PROVACI TU...

1. $\displaystyle\int \frac{1}{1-\sin x}\,dx = \cot\left(\frac{\pi}{4} - \frac{x}{2}\right) + c$

2. $\displaystyle\int \frac{1-\cos x}{1+\cos x}\,dx = -x + 2\tan\frac{x}{2} + c$