REGOLA DI DE HÔPITAL

PER LE FORME INDETERMINATE: $\left[\frac{0}{0}\right]$ $\left[\frac{\infty}{\infty}\right]$

$$\lim_{x \to x^*} \frac{f(x)}{g(x)} = \left\langle \begin{matrix} \left[\frac{0}{0}\right] \\ \left[\frac{\infty}{\infty}\right] \end{matrix} \right\rangle \overset{H}{=} \lim_{x \to x^*} \frac{f'(x)}{g'(x)}$$

$x_0 \quad \infty$

SI DERIVANO NUMERATORE E DENOMINATORE

ESERCIZI SVOLTI

EX 1 $$\lim_{x \to 2} \frac{x^4 - 5x^3 + 6x^2 + 4x - 8}{x^3 - 4x^2 + 4x}$$

EX 2 $$\lim_{x \to +\infty} \frac{e^x}{\ln x}$$

EX 3 $$\lim_{x \to 0} \frac{3^x - 2^x}{x}$$

EX 4. $$\lim_{x \to 1} \frac{\sqrt{x+1} - \sqrt{2}}{x^2 - 1}$$

EX 5 $$\lim_{x \to 0^+} \frac{\ln(\tan 3x)}{\ln(\tan x)}$$

EX 6. $$\lim_{x \to +\infty} \frac{\ln(3x^2 - x - 1)}{\ln(x^2 + x)}$$

EX 7. $$\lim_{x \to 0^+} \frac{\ln x^5}{\cot x}$$

ESERCIZI SVOLTI

1 $$\lim_{x \to 2} \frac{x^4 - 5x^3 + 6x^2 + 4x - 8}{x^3 - 4x^2 + 4x} = \left[\frac{0}{0}\right] \stackrel{H}{=}$$

SENZA DE HOPITAL, DOVREMMO SCOMPORRE NUMERATORE E DENOMINATORE, UTILIZZANDO RUFFINI

$$\stackrel{H}{=} \lim_{x \to 2} \frac{4x^3 - 15x^2 + 12x + 4}{3x^2 - 8x + 4} = \left[\frac{32 - 60 + 24 + 4}{12 - 16 + 4}\right] = \left[\frac{0}{0}\right] \stackrel{H}{=}$$

$$\stackrel{H}{=} \lim_{x \to 2} \frac{12x^2 - 30x + 12}{6x - 8} = \frac{48 - 60 + 12}{12 - 8} = \frac{0}{4} = 0$$

EX 2

$$\lim_{x\to+\infty}\frac{e^x}{\ln x}=\left[\frac{\infty}{\infty}\right]\overset{H}{=}\lim_{x\to+\infty}\frac{e^x}{\frac{1}{x}}=$$

$$=\lim_{x\to+\infty}x\,e^x=\left[+\infty\cdot+\infty\right]=+\infty$$

COSA AVREMMO FATTO SENZA DE HOPITAL?

AVREMMO UTILIZZATO IL CONFRONTO TRA GLI INFINITI

EX 3

$$\lim_{x\to0}\frac{3^x-2^x}{x}=\left[\frac{1-1}{0}\right]=\left[\frac{0}{0}\right]\overset{H}{=}$$

$$\overset{H}{=}\lim_{x\to0}\frac{3^x\ln3-2^x\ln2}{1}=\ln3-\ln2$$

COSA AVREMMO FATTO SENZA DE HOPITAL?

AVREMMO UTILIZZATO IL LIMITE NOTEVOLE

$$\lim_{x\to0}\frac{a^x-1}{x}=\ln a$$

SOTTRAENDO E ADDIZIONANDO 1

4.

$$\lim_{x\to 1}\frac{\sqrt{x+1}-\sqrt{2}}{x^2-1}=\left[\frac{0}{0}\right]\overset{H}{=}\lim_{x\to 1}\frac{\frac{1}{2\sqrt{x+1}}-0}{2x}=$$

$$=\frac{\frac{1}{2\sqrt{2}}}{2}=\frac{1}{2\sqrt{2}}\cdot\frac{1}{2}=\frac{\sqrt{2}}{8}$$

COSA AVREMMO FATTO SENZA DE HOPITAL ?

AVREMMO RAZIONALIZZATO IL NUMERATORE

EX 5

$$\lim_{x\to 0^+}\frac{\ln(\tan 3x)}{\ln(\tan x)}=\left[\frac{\ln 0^+}{\ln 0^+}\right]=\left[\frac{\infty}{\infty}\right]\overset{H}{=}$$

$$\overset{H}{=}\lim_{x\to 0^+}\frac{\frac{1}{\tan 3x}\cdot\underbrace{(1+\tan^2 3x)}_{\to 1}\cdot 3}{\frac{1}{\tan x}\cdot\underbrace{(1+\tan^2 x)}_{\to 1}}=$$

$$=3\cdot\lim_{x\to 0^+}\frac{\tan x}{\tan 3x}=\left[\frac{0}{0}\right]\overset{H}{=}$$

$$\overset{H}{=}3\lim_{x\to 0^+}\frac{1+\tan^2 x}{(1+\tan^2 3x)\cdot 3}=3\cdot\frac{1}{3}=1$$

EX 6.

$$\lim_{x\to+\infty}\frac{\ln(3x^2-x-1)}{\ln(x^2+x)} = \left[\frac{\infty}{\infty}\right] \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to+\infty}\frac{\dfrac{6x-1}{3x^2-x-1}}{\dfrac{2x+1}{x^2+x}} =$$

$$= \lim_{x\to+\infty}\frac{6x-1}{3x^2-x-1}\cdot\frac{x^2+x}{2x+1} = \lim_{x\to+\infty}\frac{6x^3}{6x^3} = 1$$

EX 7.

$$\lim_{x\to 0^+}\frac{\ln x^5}{\cot x} = \left[\frac{\infty}{\infty}\right] \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to 0^+}\frac{\dfrac{5\cancel{x^4}}{\cancel{x^5}}}{-\dfrac{1}{\sin^2 x}} = \lim_{x\to 0^+} -\frac{5\sin^2 x}{x} = \left[\frac{0}{0}\right] \overset{H}{=}$$

$\sin 0 = 0$

$$\overset{H}{=} \lim_{x\to 0^+} -\frac{10\sin x\cos x}{1} = 0$$

PROVACI TU...

1. $$\lim_{x \to 1} \frac{x^4 - 2x^3 + 2x^2 - 2x + 1}{x^4 - 2x^3 + x^2} = 2$$

2. $$\lim_{x \to 1} \frac{\sqrt[3]{2x+6} - \sqrt[3]{8}}{x-1} = \frac{1}{6}$$

3. $$\lim_{x \to 0^-} \frac{\tan x}{\cos x - 1} = +\infty$$

4. $$\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x} = 2$$

5. $$\lim_{x \to +\infty} \frac{x^4 + 1}{\ln^2 x + \ln x} = +\infty$$

6. $$\lim_{x \to \pi} \frac{\tan^2 x/2}{\ln(\pi - x)^2} = -\infty$$

7. $$\lim_{x \to 2^-} \frac{e^{\frac{1}{2-x}}}{\ln(2-x)} = -\infty$$

8. $$\lim_{x \to 0^+} \frac{\ln x}{\cot x} = 0$$

9. $$\lim_{x \to 0} \frac{\arctan x}{x^2} = \infty$$

FORME INDETERMINATE

$[+\infty - \infty]$ $[0 \cdot \infty]$ $[0^0]$ $[1^\infty]$ $[\infty^0]$

ESERCIZI SVOLTI

EX 1 $\lim\limits_{x \to +\infty} (\ln x - x)$

EX 2 $\lim\limits_{x \to 0^+} x \cdot \ln x$

EX 3 $\lim\limits_{x \to +\infty} \left[x\left(e^{\frac{1}{x}} - 1\right)\right]$

EX 4 $\lim\limits_{x \to -1^-} (x^2 - 1)^{x^2 - 2x - 3}$

EX 5 $\lim\limits_{x \to 0} (\cos x)^{\frac{2}{x^2}}$

EX 6 $\lim\limits_{x \to 0^+} (\sin x)^{\tan x}$

FORMA INDETERMINATA $[+\infty - \infty]$

SI RICONDUCE A $\left[\frac{\infty}{\infty}\right]$

ESEMPI

1. $$\lim_{x \to +\infty} (\ln x - x) = [+\infty - \infty]$$

$$= \lim_{x \to +\infty} x\left(\underbrace{\frac{\ln x}{x}}_{\to 0} - 1\right) \overset{*}{=} \lim_{x \to +\infty} -x = -\infty$$

X IN EVIDENZA

$*$

$$\lim_{x \to +\infty} \frac{\ln x}{x} \overset{H}{=} \lim_{x \to +\infty} \frac{\frac{1}{x}}{1} = 0 \quad *$$

FORMA $[0 \cdot \infty]$

$$a \cdot b = \frac{b}{\frac{1}{a}}$$

3. $$\lim_{x \to 0^+} x \cdot \ln x = [0^+ \cdot -\infty]$$

$$= \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} = \left[\frac{\infty}{\infty}\right] \overset{H}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} =$$

$$= \lim_{x \to 0^+} \frac{-x^2}{x} = \lim_{x \to 0^+} -x = 0^-$$

3. $$\lim_{x\to+\infty}\left[x\left(e^{\frac{1}{x}}-1\right)\right] = [0\cdot\infty]$$

$$= \lim_{x\to+\infty}\frac{e^{\frac{1}{x}}-1}{\frac{1}{x}} = \left[\frac{0}{0}\right] \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to+\infty}\frac{e^{\frac{1}{x}}\cdot\cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{-\frac{1}{x^2}}} = e^{0^+} = 1$$

PROVACI TU...

1. $$\lim_{x\to2^+}(x-2)\ln(x-2) = 0$$

2. $$\lim_{x\to+\infty}\left(x\cdot e^{-x^2}\right) = 0$$

3. $$\lim_{x\to+\infty} x^2\cdot e^{-x} = 0$$

4. $$\lim_{x\to+\infty}\frac{1}{x}\ln\frac{e^x-1}{x} = 1$$

FORME INDETERMINATE

$$\left[0^0\right] \quad \left[1^\infty\right] \quad \left[\infty^0\right]$$

SI RICONDUCONO SEMPRE ALLE FORME $\left[\frac{0}{0}\right]$ $\left[\frac{\infty}{\infty}\right]$

UTILIZZANDO LA REGOLA:

$$\boxed{f(x)^{g(x)} = e^{g(x)\ln f(x)} = e^{\frac{\ln f(x)}{\frac{1}{g(x)}}}}$$

$a \cdot b = \frac{b}{\frac{1}{a}}$

ESEMPI

EX 1 $\displaystyle\lim_{x\to -1^-} \left(x^2-1\right)^{x^2-2x-3} = \left[0^0\right]$

$$= \lim_{x\to -1^-} e^{\left(x^2-2x-3\right)\ln\left(x^2-1\right)} = e^{\left[0\cdot\infty\right]}$$

$$= \lim_{x\to -1^-} e^{\frac{\ln\left(x^2-1\right)}{\frac{1}{x^2-2x-3}}} = e^{\left[\frac{\infty}{\infty}\right]} \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to -1^-} e^{\frac{\frac{2x}{(x^2-1)}}{\frac{0-(2x-2)}{(x^2-2x-3)^2}}} = \lim_{x\to -1^-} e^{\frac{2x}{(x^2-1)}\cdot\frac{(x^2-2x-3)^2}{-(2x-2)}} =$$

FRAZIONE DI FRAZIONE

$$= e^{\left[\frac{0}{0}\right]}$$

SE VOGLIAMO APPLICARE DE HOPITAL OCCORRE PRIMA MOLTIPLICARE

$$= \lim_{x\to -1^-} e^{\frac{(x^4-2x^3-3x^2)^2}{-x^3+x^2+x}} =$$

ENTRANDO X E' PIU' FACILE DERIVARE

TROPPO COMPLICATO!!

$$\overset{H}{=} \lim_{x\to -1^-} e^{\frac{2(x^4-2x^3-3x^2)^2\cdot(4x^3-6x^2-6x)}{-3x^2+2x+1}} = e^{\left[\frac{0}{0}\right]}$$

E' MEGLIO LAVORARE DIVERSAMENTE..

SE RICONSIDERIAMO IL NOSTRO LIMITE..

$(x+1)(x-3)$

$$\lim_{x\to -1^-} e^{\frac{2x}{(x^2-1)}\cdot\frac{(x^2-2x-3)^2}{-(2x-2)}} =$$

METTO IN EVIDENZA IL 2 E SEMPLIFICO

$$= \lim_{x\to -1^-} e^{\frac{x\,(x+1)^{\cancel{2}}(x-3)^2}{-\cancel{(x+1)}(x-1)(x-1)}} = e^{\frac{0}{4}} = e^0 = 1$$

EX2

$$\lim_{x\to 0} (\cos x)^{\frac{2}{x^2}} = \left[1^{\infty}\right] =$$

$$= \lim_{x\to 0} e^{\frac{2}{x^2}\ln(\cos x)} = \lim_{x\to 0} e^{\frac{2\ln(\cos x)}{x^2}} = e^{\left[\frac{0}{0}\right]} \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to 0} e^{\frac{\cancel{2}\,\frac{-\sin x}{\cos x}}{\cancel{2}x}} = \lim_{x\to 0} e^{\frac{-\tan x}{x}} = e^{\left[\frac{0}{0}\right]} \overset{H}{=}$$

$\tan x$

$$D\tan x = \left(1+\tan^2 x\right)$$

$$(\tan 0 = 0)$$

$$\overset{H}{=} \lim_{x\to 0} e^{\frac{-\left(1+\tan^2 x\right)}{1}} = e^{-1} = \frac{1}{e}$$

EX 3

$$\lim_{x\to\pi^-}\left(\frac{1}{\sin x}\right)^{\pi-x} = [\infty^0] =$$

$\sin\pi^- = 0^+$

$$f(x)^{g(x)} = e^{g(x)\ln f(x)}$$

$$= \lim_{x\to\pi^-} e^{(\pi-x)\ln\left(\frac{1}{\sin x}\right)} = \lim_{x\to\pi^-} e^{-(\pi-x)\ln\sin x} =$$

OSS. $\ln\frac{1}{a} = \ln 1 - \ln a = 0 - \ln a = -\ln a$

$$= \lim_{x\to\pi^-} e^{\frac{\ln\sin x}{\frac{1}{(x-\pi)}}} = e^{\left[\frac{\infty}{\infty}\right]} \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to\pi^-} e^{\frac{\frac{\cos x}{\sin x}}{\frac{-1}{(x-\pi)^2}}} = \lim_{x\to\pi^-} e^{-\cot x\,(x-\pi)^2} =$$

$\frac{\cos x}{\sin x} \leftarrow \cot x$

$\cot x = \frac{1}{\tan x}$

$$= \lim_{x\to\pi^-} e^{-\frac{(x-\pi)^2}{\tan x}} = e^{\left[\frac{0}{0}\right]} \overset{H}{=}$$

$\tan\pi = 0$

$$\overset{H}{=} \lim_{x\to\pi^-} e^{-\frac{2(x-\pi)}{1+\tan^2 x}} = e^{\frac{0}{1}} = e^0 = 1$$

EX 4

$$\lim_{x\to 0^+} (\sin x)^{\tan x} = \left[0^0\right] =$$

$$= \lim_{x\to 0^+} e^{\tan x \cdot \ln(\sin x)} = \lim_{x\to 0^+} e^{\frac{\ln(\sin x)}{\cot x}} = e^{\left[\frac{\infty}{\infty}\right]} \overset{H}{=}$$

CONVIENE TRASFORMARE: $\tan x = \dfrac{1}{\cot x}$

$\cot 0^+ = +\infty$

$\tan 0^+ = 0^+$

$$\overset{H}{=} \lim_{x\to 0^+} e^{\frac{\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}}{-\frac{1}{\sin^2 x}}} =$$

$$\frac{-\sin x}{\cos x} \leftarrow \tan x$$

$$D\cot x = -(1+\cot^2 x) = -\frac{1}{\sin^2 x}$$

$$= \lim_{x\to 0^+} e^{-\tan x \sin^2 x} = e^0 = 1$$

PROVACI TU

1. $\lim_{x \to 0} (1-2x)^{\frac{1}{x}} = \frac{1}{e^2}$

2. $\lim_{x \to 1^+} (\ln x)^{1-x} = 1$

3. $\lim_{x \to 0^-} (-x)^{x} = 1$

4. $\lim_{x \to \left(\frac{\pi}{2}\right)^-} (\tan x)^{\cot x} = 1$