m DISPARI

NON SI IMPONGONO CONDIZIONI DI ESISTENZA

IN OGNI CASO SI ELEVA ALL'INDICE DELLA RADICE

$$\sqrt[2m+1]{A(x)} \gtrless B(x) \Rightarrow A(x) \gtrless B(x)^{2m+1}$$

ESEMPIO 1

$$\left(\cancel{\sqrt[3]{1-x}}\right)^{\cancel{3}} < (-2)^3 \Rightarrow 1-x < -8 \Rightarrow x > 9$$

EX 2

$$\left(\cancel{\sqrt[3]{x^3-2x+1}}\right)^{\cancel{3}} \geq (x)^3 \Rightarrow \cancel{x^3} - 2x + 1 \geq \cancel{x^3}$$

$$S: \quad x \leq \frac{1}{2}$$

EX 3 $\left(\cancel{\sqrt[5]{x^5-32}}\right)^{\cancel{5}} < (0)^5 \Rightarrow x^5 - 32 < 0 \Rightarrow x < 2$

EX 4 (CON IL CUBO DI BINOMIO)

$$\sqrt[3]{x^3-6x^2+2x-1} \leq x-2 \quad \Rightarrow$$

$$x^3-6x^2+2x-1 \leq (x-2)^3$$

$$(A \mp B)^3 = A^3 \mp 3A^2B + 3AB^2 \mp B^3$$

$$\Rightarrow \cancel{x^3}-\cancel{6x^2}+2x-1 \leq \cancel{x^3}-\cancel{6x^2}+12x-8$$

$$S: \quad x \geq \frac{7}{10}$$

PROVACI TU...

1) $\sqrt[3]{x^2-4} \leq 0$

2) $\sqrt[3]{2x-3} > -2$

3) $\sqrt[3]{x^3-5x} \leq x$ $\qquad S: \; x \leq 0$

4) $\sqrt[3]{x^3+9x^2+27} > x+3$ $\qquad S: \; x < 0$

5) $\sqrt[5]{x-1} \geq \sqrt[5]{2x+1}$

DISEQUAZIONI IRRAZIONALI
CON VALORI ASSOLUTI

PRIMO CASO $\quad \sqrt{|A(x)|} \gtrless B(x)$

A) $\sqrt{|A(x)|} < B(x)$

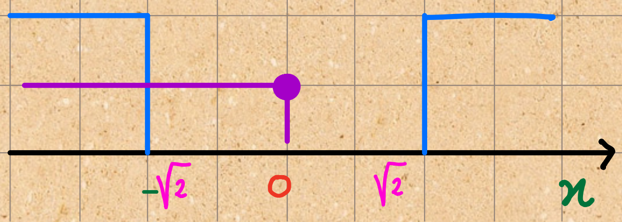
EX $\quad \sqrt{|x^2-4|} < -x$

$$\begin{cases} |x^2-4| \ge 0 & \Longrightarrow \forall x \in \mathbb{R} \\ -x \ge 0 & \Longrightarrow x \le 0 \\ |x^2-4| < \underbrace{x^2}_{+K} & \text{VALORI INTERNI } * \end{cases}$$

$* \quad -x^2 < x^2-4 < x^2 \qquad\qquad 4-x^2 < x^2 < 4+x^2$

$$\begin{cases} x^2 > 4-x^2 \\ \cancel{x^2} < 4+\cancel{x^2} \end{cases} \qquad \begin{cases} \cancel{2}x^2 > \cancel{4}^{\,2} \\ \forall x \in \mathbb{R} \end{cases} \qquad x \le -\sqrt{2} \vee x \ge +\sqrt{2}$$

S: $x < -\sqrt{2}$



B $\sqrt{|A(x)|} > B(x)$

EX. $\sqrt{|x^2-x|} > -2x$

$$1\begin{cases} |x^2-x| \geq 0 \\ -2x < 0 \end{cases} \cup \quad 2\begin{cases} -2x \geq 0 \\ |x^2-x| > \underbrace{4x^2}_{+} \end{cases}$$

VAL. ESTERNI

$$1\begin{cases} \forall x \in \mathbb{R} \\ x > 0 \end{cases} \cup \quad 2\begin{cases} x \leq 0 \\ x^2-x < -4x^2 \vee x^2-x > 4x^2 \; * \end{cases}$$

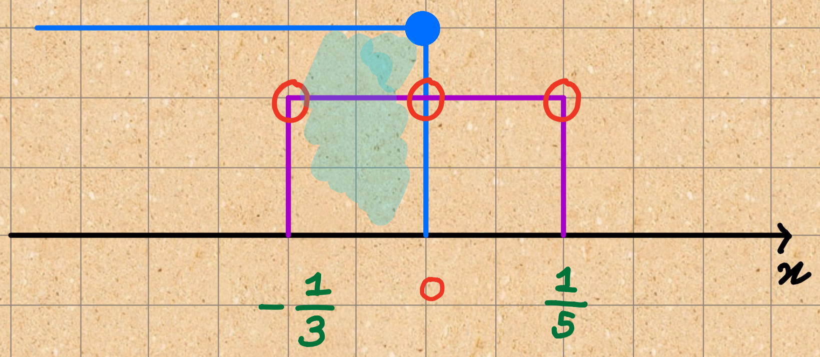
$S_1: x > 0$

$* \quad 5x^2-x < 0 \vee 3x^2+x < 0$

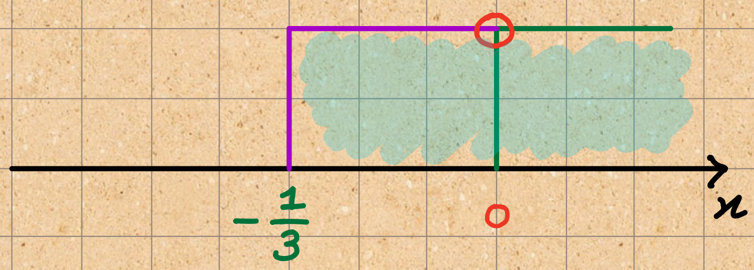
$$\underbrace{0 < x < \frac{1}{5} \vee -\frac{1}{3} < x < 0}$$

$S_*: \quad -\frac{1}{3} < x < \frac{1}{5} \wedge x \neq 0$

$S: -\frac{1}{3} < x < 0$



$S_1 \cup S_2$



$S_{tot}: x > -\frac{1}{3} \wedge x \neq 0$

SECONDO CASO $\sqrt{A(x)} \gtrless \underbrace{|B(x)|}_{+}$

DOPO LA C.E. SI ELEVA AL QUADRATO

Ex 1

$$\sqrt{x^2+x} < \underbrace{|2x|}_{+}$$

$$\begin{cases} x^2+x \geq 0 \\ x^2+x < 4x^2 \end{cases} \Rightarrow \begin{cases} \\ \end{cases}$$

$$\left(|a|\right)^2 = a^2$$

S:

EX 2 $\sqrt{x^2-3x} \geq \underbrace{|-2x|}_{+K}$

BASTA ELEVARE AL QUADRATO

$$x^2-3x \geq 4x^2 \Rightarrow$$

S:

TERZO CASO $\sqrt{\underbrace{|A(x)|}_{+}} \gtrless \underbrace{|B(x)|}_{+}$

SI ELEVA DIRETTAMENTE AL QUADRATO

EX $\sqrt{|x^2-x|} < |2x|$

$\Rightarrow \quad |x^2-x| < \underbrace{4x^2}_{+}$ VAL. INTERNI

$$-4x^2 < x^2-x < +4x^2$$

$$\begin{cases} x^2-x > -4x^2 \\ x^2-x < 4x^2 \end{cases} \qquad \begin{cases} 5x-x^2 > 0 \\ 3x+x^2 > 0 \end{cases}$$

$$\begin{cases} x < 0 \ \vee \ x > \frac{1}{5} \\ x < -\frac{1}{3} \ \vee \ x > 0 \end{cases} \qquad x < -\frac{1}{3} \ \vee \ x > \frac{1}{5}$$

PROVACI TU...

1) $\sqrt{|3x-x^2|} < -3x$

2) $\sqrt{|3x-x^2|} \geq 3x$

3) $\sqrt{3x-x^2} \leq |-3x|$

4) $\sqrt{3x-x^2} > |3x|$

5) $\sqrt{|3x-x^2|} < |3x|$

6) $\sqrt{|3x-x^2|} \geq |-3x|$

SFIDE

1) $\underbrace{\sqrt{|x^2-1|}}_{+} \leq \underbrace{-|x+1|}_{-}$ $\qquad x = -1$

2) $\underbrace{\sqrt{|x^2-1|}}_{+} > \underbrace{-|x+1|}_{-}$ $\qquad x \neq -1$

3) $\sqrt{\underset{\text{C.E.}}{x^2-1}} \geq \underbrace{-|x+1|}_{-}$ $\qquad x \leq -1 \vee x \geq +1$

4) $\sqrt{\underset{\text{C.E.}}{x^2-1}} > \underbrace{-|x+1|}_{-}$ $\qquad x < -1 \vee x \geq +1$

5) $\sqrt{\underset{\text{C.E.}}{x^2-1}} > \underbrace{-|x+3|}_{-}$ $\qquad x < -1 \vee x > +1$